CAT 1: Numerical Methods for Engineers (MAT 2263)
Year 2 SGE
Duration: 1 hour 30 minutes

1. Find a zero of $f(x) = 56x^2 + 5x - 6$ correct to one decimal place using the bisection method, then find a bound of the error in the form $|\text{exact value} - \text{approximation}| < 0.5 \cdot 10^{-k}$ and conclude. (Hint: first find the exact values of the zeros of $f(x)$, determine which you have approximated, then proceed). **(6 points)**

2. Let $f(x) = 7x^4 + 2x^3 - 4000$. Use the fixed-point iteration method to find a zero of $f(x)$ correct to 4 decimal places. **(5 points)**

3. Derive the normal equations **(4 points)**

$$\begin{cases} \sum yx &= a\sum x^2 + b\sum x \\ \sum y &= a\sum x + b\sum 1 \end{cases}$$

for finding the best fit line $y = ax + b$ for given data:

x	x_1	$\cdots$	x_n
y	y_1	$\cdots$	y_n

4. Find the best fit parabola for the following data: **(5 points)**

x	1	2	3
y	6	8	16

5. Consider the function $y = y(x)$ given by the data:

x	1.0	1.2	1.4	1.6
y	3.25	5.32	8.07	11.56

   (a) Find the interpolating polynomial using Gregory-Newton forward interpolation formula; **(2 points)**

   (b) Estimate y when $x = 1.25$; **(1 point)**

   (c) Estimate x with four decimals when $y = 10$; **(2 points)**

6. Use Newton's divided differences interpolation formula to find an approximation of y corresponding to $x = 6.8$: **(5 points)**

x	-1	2	7	8
y	-12	-9	36	78

UNIVERSITY of RWANDA

COLLEGE OF SCIENCE AND TECHNOLOGY

COLLEGE EXAMINATIONS – ACADEMIC YEAR 2024- 2025

SCHOOL OF SCIENCE

DEPARTMENT OF MATHEMATICS

SECOND YEAR SEMESTER II

FINAL EXAMINATION

NUMERICAL METHODS FOR ENGINEERS (MAT2263)

DATE: ....../June/2025 **TIME: 2 hours**

MAXIMUM MARKS = 50

INSTRUCTIONS:

1. This paper contains **TWO** sections.
2. Section A is compulsory, and Answer any **TWO** of the **THREE** questions in Section B.
3. No written materials allowed into the Examination Room.
4. Write all your answers in the answer booklet provided.
5. Do not forget to write your Registration Number.
6. Do not write any answers on this question paper.
7. Where appropriate draw large clearly labeled diagrams in your answers.

SECTION A: ATTEMPT ALL QUESTIONS /20 marks

QUESTION 1./20 marks

a) By Newton-Raphson method to get the approximation of the root of the equation $f(x) = 4\cos(x) - 5\sin(x) + 1 = 0$ for $x \in [-\frac{\pi}{4}, \frac{\pi}{3}]$; what is the value of the first iteration x_1? **(2 marks)**

b) Consider the following table of data.

$\sum x = 15$	$\sum y = 204$	$\sum x^2 = 55$	$\sum xy = 748$	$n = 5$

By the method of least squares, find the straight line that best fits these data, then find $y(0.5)$. **(2 marks)**

c) Using Newton's divided difference formula, find the interpolating cubic polynomial and $y(1.5)$ from the following table of values: **(3 marks)**

x	0	2	3	5
y	1	15	40	156

d) From the following table: **(3 marks)**

x	0	2	4	6
y	1	15	85	48

Find:

i) The interpolating polynomial using the backward Gregory-Newton interpolation formula;

ii) $y(2.25)$

e) From the flowing table: **(4 marks)**

x	1.0	1.2	1.4	1.6
y	2.7183	3.3201	4.0552	4.9530

i) Find $\frac{dy}{dx}$ $at\ x = 1.2$.

ii) By Simpson's 3/8 rule find $\int_{1.0}^{1.6} y\,dx$.

f) Write down the algorithm to Runge - Kutta method of fourth order and the formula of GAUSS - SEIDEL Iteration Method for getting x_1, y_1 and z_1.

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(3 marks)

g) A furniture maker has 6units of wood and 28hours of free time, in which he will make decorative screens. Two models have sold well in the past, so he restricts himself to those. He estimates that model I requires 2 units of woods and 7hours of time, while model II requires 1 unit of woods and 8hours of time. The prices of the models are US \$ 120 and US \$80 respectively. The furniture maker wants to maximize the revenue from the sale of the decorative screens.

Formulate this situation in mathematical model? **(3 marks)**

SECTION B: ATTEMTY ONLY TWO QUESTIONS /30 marks

QUESTION 2.

a. Calculate the first and second derivatives of the function tabulated in the difference table at the point $x = 1.8$ **(10 marks)**

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.0 x_0	2.7183				
		0.6018			
1.2 x_1	3.3201		0.1333		
		0.7351		0.0067	
1.4 x_2	4.0552		0.1627		0.0013
		0.8978		0.0080	
1.6 x_3	4.9530		0.1988		
		1.0966			
1.8 x_4	6.0496				

b. Find the missing terms in the following: **(5 marks)**

x	0	5	10	15	20	25	30
y	1	3	?	73	225	?	1153

QUESTION 3.

a. Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given as below: **(7 marks)**

x	5	6	9	11
y	12	13	14	16

b. A factory makes 3types of products P_1, P_2, P_3.The factory needs 5 units of volume to store one unit of P_1,3.5 units of volume to store one unit of P_2,4.2 units of volume to store one unit of P_3 The factory uses 3 hours of manpower to make 1 unit of P_1,4 hours of manpower to make 1 unit of P_2,6 hours of manpower to make 1 unit of P_3

It sells 1 unit of P_1 at 450 Frw, 1 unit of P_2 at 620 Frw, 1 unit of P_3 at 780 Frw. The factory has only 40 units of stock volume, it cannot pay over 20 hours of manpower. The factory wishes to maximize the revenue it would make from the production and sale of its products. Formulate it as the optimization LPP. **(8 marks)**

QUESTION 4.

a. Use the fourth order Runge-Kutta method, Compute an estimate of $x(1.1)$ for the initial- value problem $\frac{dx}{dt} = \frac{1}{x+t}, x(1) = 2$ using step size $h = 0.1$, with six decimal places.

(10 marks)

b. Find the cubic polynomial which takes the following values: $y(1) = 24, y(3) = 120, y(5) = 336$ and $y(7) = 720$. Hence, obtain the value of $y(8)$.

(5 marks)

END ASSESSMENT!