



CAT of "Probability and Statistics for Engineers"

Duration: 1.5 Hrs

Instructions: Attempt ALL Questions

Q1. The following table gives a sample of energy consumption of households in a certain area:

Energy (kWh)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of Households	120	184	124	95	85	70	60	44	21	17

(a) Find:

(i) The sample mean /1.5 Pts

(ii) The sample Standard deviation /1.5 Pts

(iii) The sample Mode /1.5 Pts

(b) Calculate the Kurtosis and interpret the results /1.5 Pts

(c) Represent the data on a Histogram /1 Pt

Q2. Let A and B be any events. If $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$.

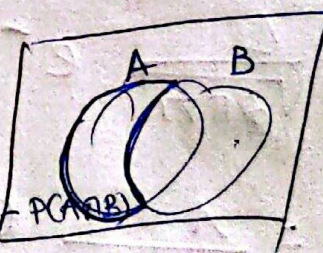
Calculate:

(a) $P(A')$ /0.5 Pts $P(A') = 1 - P(A)$

(b) $P(B')$ /0.5 Pts $P(B') = 1 - P(B)$

(c) $P(A \cup B)$ /0.5 Pts $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(d) $P(A' \cap B')$ /0.5 Pts $P(A' \cap B') = 1 - P(A \cup B)$



$$\frac{820}{2} + \frac{820}{2} + 1$$

$$410 + 411$$

Q3. ENERGY CO. LTD operates two types of sensors (Sensor A & Sensor B) in a power plant to monitor temperature. Sensor A has 5% failure rate while Sensor B has 4% failure rate. Sensor A is used 70% of the time while sensor B is used 30% of the time. If a randomly selected sensor indicates a failure, what is the probability that it was Sensor A? /2.5 Pts



UNIVERSITY of
RWANDA

COLLEGE OF SCIENCE AND
TECHNOLOGY

End of First Semester Final Examination

Academic Year: 2024

Module: Probability & Statistics for Engineers (MAT2165)

Candidates: Year Two: MEE, ENE, CEGE-CE, CEGE-WREE, CEGE-TRE, CEGE-SGE

Period: 2 Hours

Maximum Marks: 50

Date: .../01/2025

INSTRUCTIONS:

1. This question paper contains two sections A and B.
2. Section A is compulsory and chooses 2 questions in section B.
3. Section A carries 20 marks.
4. Section B carries 30 marks and each question carries 15 marks
5. Start new question from a fresh page.
6. Do not forget to write your Registration Number.
7. Write all your answers in the booklet provided.
8. Do not write any answers on this question paper.
9. Calculators are allowed.
10. Statistical distribution tables are attached

SECTION A (Compulsory) /20 Marks

Question I

- (a) The article "A Thin-Film Oxygen Uptake Test for the Evaluation of Automotive Crankcase Lubricants" (Lubric. Engr., 1984: 75-83) reported the following data on oxidation-induction time (min) for various commercial oils: 87, 103, 130, 160, 180, 195, 132, 145, 211, 105, 145, 153, 152, 138, 87, 99, 93, 119, 129. Compute the mean, median, mode and plot the cumulative frequency polygon of oxidation-induction time. /4 Marks
- (b) It is known that of the articles produced by a factory, 20% come from Machine A, 30% from Machine B, and 50% from Machine C. The percentages of satisfactory articles among those produced are 95% for A, 85% for B and 90% for C. An article is chosen at random.
- What is the probability that it is satisfactory? /2 Marks
 - Assuming that the article is satisfactory, what is the probability that it was produced by Machine A? /2 Marks
- (c) Truck tires are tested over rough terrain. 25% of the trucks fail to complete the test run without a blowout. Of the next fifteen trucks through the test,
- Find the probability that at least two tires have blowouts /3 Marks
 - What would be the expected number and standard deviation of the number of trucks with blowouts of the next fifteen tested? /2 Marks
- (d) STEEL Co. Ltd. manufactures steel rods for construction. To verify the quality of their rods, a sample of 25 rods was selected. Their mean and standard deviation are 12 cm and 0.2 cm, respectively. Find the two-sided tolerance limits that cover 90% of the rods with a confidence level of 95% to ensure the rods meet the design specifications. /3 Marks
- (e) The following data relate x , the moisture of a wet mix of a certain product, to Y the density of the finished product.

x_i	5	6	7	10	12	15	18	20
Y_i	7.4	9.3	10.6	15.4	18.1	22.2	24.1	24.8

The following table summarizes the results output generated by statistical software:

SUMMARY OUTPUT					
Regression Statistics					
Multiple R	0.985651195				
R Square	0.971508279				
Adjusted R Square	0.966759659				
Standard Error	1.256301301				
Observations	8				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>gnificance F</i>
Regression	1	322.8989923	322.899	204.5875	7.31E-06
Residual	6	9.469757746	1.578293		
Total	7	332.36875			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%Upper 95%ower 95.0pper 95.0</i>
Intercept	2.463492958	1.076383021	2.288677	0.062057	-0.17032 5.097307 -0.17032 5.097307
xi	1.206366197	0.084341171	14.30341	7.31E-06	0.999991 1.412742 0.999991 1.412742

- Use the above summary results to write-down a linear regression model that fits the data.
/2 Marks
- What would be the value of the density of the finished product if the moisture of a wet mix of the product was 17? /2 Marks

SECTION B (Choose ONLY two Questions) /30 Marks

Question II:

- (a) Given the following data :

Classes	[410-420[	[420-430[	[430-440[	[440-450[	[450-460[	[460-470[	[470-480[
Frequency	14	20	42	54	45	18	7

Find :

- Mean /2 Marks
 - Median /2 Marks
 - Mode /2 Marks
 - Kurtosis coefficient and interpret it /3 Marks
- (b) An experiment was carried out to compare electrical resistivity for six different low permeability concrete bridge deck mixtures. There were 26 measurements on concrete cylinders for each mixture; these were obtained 28 days after casting. The entries in the accompanying ANOVA table are based on information in the article "In-Place Resistivity of Bridge Deck Concrete Mixtures" (ACI Materials J.,2009: 114-122). Fill in the missing entries and test appropriate

hypotheses (use a significance level of 5%). From your conclusion, is it necessary to perform multiple pairwise comparison tests? If yes, why? If no, why? /6Marks

ANOVA				
Source of Variation	Sum Squares	Degree of Freedom	Mean Squares	F
Between groups	*	*	*	
Within groups	*	*	13.929	*
Total	5664.415	*		

Question III:

- (a) A substance used in a research is shipped by airfreight to users in cartons of 1000 ampules. The data below, involving 10 shipments were collected on the number of times the carton was transferred from one aircraft to another over the shipment route (X) and the number of ampules found to be broken upon arrival (Y). Assume that a simple regression model is appropriate.

i	1	2	3	4	5	6	7	8	9	10
X_i	1	0	2	0	3	1	0	1	2	0
Y_i	16	9	17	12	22	13	8	15	19	11

- Use least squares method to obtain the fitted (estimated) regression model. /5 Marks
 - Obtain a point estimate of the expected number of broken ampules when $X = 1$ transfer is made. /2 Marks
 - Determine and interpret the correlation coefficient /2 Marks
- (b) Nowadays, electric cars manufacturers are in competition, especially, in optimizing the travel distance of fully-charged cars. One of the manufacturers claims that their car can travel an average distance of at least 600 km when is fully charged. To verify this claim, a study on 40 randomly selected cars is conducted. If the results indicate that the cars can travel 580 km on average, with a standard deviation of 30 km, test the manufacturer's claim at 5% level of significance. /6 Marks

Question IV:

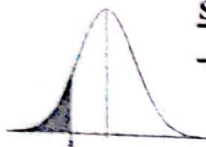
- (a) The following numbers are observations on tensile strength of synthetic fabric specimens selected from a production process at equally spaced time intervals:

Sample No.	Tensile Strength			
1	51.3	51.7	49.5	50.1
2	51.0	50.3	49.3	50.0
3	50.8	51.1	49.0	49.4
4	50.6	51.1	49.2	50.1
5	49.6	50.5	50.9	51.0
6	51.3	52.0	50.3	51.2
7	49.7	50.5	50.4	50.2
8	51.8	50.3	50.0	50.5
9	48.6	50.3	51.0	49.2
10	49.6	49.8	50.5	50.1

- Construct $\bar{X}$ -chart /4 Marks
 - Construct R-chart /4 Marks
 - For each chart in *i* and *ii* above, determine if the process is in-control or out-of-control. /2 Marks
- (b) The lifetime (in years) of one component of a projector follows a Gamma distribution with expected value 4 and variance 16. What is the probability that this component has a lifetime of at least 8 years? /5 Marks



Standard Normal Distribution Tables



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

F Distribution Table

d.f.D.	$\alpha = 0.05$																			
	d.f.N.																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.72	2.65	2.61	2.57	2.53	2.54	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.10	2.06	2.01	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.07	2.02	1.97	1.92	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.12	2.08	2.04	1.99	1.95	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.13	2.09	2.05	2.01	1.96	1.91	1.84	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.06	2.02	1.98	1.93	1.89	1.81	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.99	1.94	1.89	1.84	1.78	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.12	2.04	2.00	1.96	1.91	1.86	1.81	1.76	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.10	2.02	1.98	1.94	1.89	1.84	1.79	1.73	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.23	2.16	2.08	2.00	1.96	1.91	1.86	1.81	1.76	1.71	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.05	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.17	2.10	2.02	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

■ APPENDIX VII

Factors for Two-Sided Normal Tolerance Limits

n	90% Confidence That Percentage of Population Between Limits Is			95% Confidence That Percentage of Population Between Limits Is			99% Confidence That Percentage of Population Between Limits Is		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
2	15.98	18.80	24.17	32.02	37.67	48.43	160.2	188.5	242.3
3	5.847	6.919	8.974	8.380	9.916	12.86	18.93	22.40	29.06
4	4.166	4.943	6.440	5.369	6.370	8.299	9.398	11.15	14.53
5	3.494	4.152	5.423	4.275	5.079	6.634	6.612	7.855	10.26
6	3.131	3.723	4.870	3.712	4.414	5.775	5.337	6.345	8.301
7	2.902	3.452	4.521	3.369	4.007	5.248	4.613	5.448	7.187
8	2.743	3.264	4.278	3.136	3.732	4.891	4.147	4.936	6.468
9	2.626	3.125	4.098	2.967	3.532	4.631	3.822	4.550	5.966
10	2.535	3.018	3.959	2.829	3.379	4.433	3.582	4.265	5.594
11	2.463	2.933	3.849	2.737	3.259	4.277	3.397	4.045	5.308
12	2.404	2.863	3.758	2.655	3.162	4.150	3.250	3.870	5.079
13	2.355	2.805	3.682	2.587	3.081	4.044	3.130	3.727	4.893
14	2.314	2.756	3.618	2.529	3.012	3.955	3.029	3.608	4.737
15	2.278	2.713	3.562	2.480	2.954	3.878	2.945	3.507	4.605
16	2.246	2.676	3.514	2.437	2.903	3.812	2.872	3.421	4.492
17	2.219	2.643	3.471	2.400	2.858	3.754	2.808	3.345	4.393
18	2.194	2.614	3.433	2.366	2.819	3.702	2.753	3.279	4.307
19	2.172	2.588	3.399	2.337	2.784	3.656	2.703	3.221	4.230
20	2.152	2.564	3.368	2.310	2.752	3.615	2.659	3.168	4.161
21	2.135	2.543	3.340	2.286	2.723	3.577	2.620	3.121	4.100
22	2.118	2.524	3.315	2.264	2.697	3.543	2.584	3.078	4.044
23	2.103	2.506	3.292	2.244	2.673	3.512	2.551	3.040	3.993
24	2.089	2.489	3.270	2.225	2.651	3.483	2.522	3.004	3.947
25	2.077	2.474	3.251	2.208	2.631	3.457	2.494	2.972	3.904
26	2.065	2.460	3.232	2.193	2.612	3.432	2.469	2.941	3.865
27	2.054	2.447	3.215	2.178	2.595	3.409	2.446	2.914	3.828
28	2.044	2.435	3.199	2.164	2.579	3.388	2.424	2.888	3.794
29	2.034	2.424	3.184	2.152	2.554	3.368	2.404	2.864	3.763
30	2.025	2.413	3.170	2.140	2.549	3.350	2.385	2.841	3.733
35	1.988	2.368	3.112	2.090	2.490	3.272	2.306	2.748	3.611
40	1.959	2.334	3.066	2.052	2.445	3.213	2.247	2.677	3.518
50	1.916	2.284	3.001	1.996	2.379	3.126	2.162	2.576	3.385
60	1.887	2.248	2.955	1.958	2.333	3.066	2.103	2.506	3.293
80	1.848	2.202	2.894	1.907	2.272	2.986	2.026	2.414	3.173
100	1.822	2.172	2.854	1.874	2.233	2.934	1.977	2.355	3.096
200	1.764	2.102	2.762	1.798	2.143	2.816	1.865	2.222	2.921
500	1.717	2.046	2.689	1.737	2.070	2.721	1.777	2.117	2.783
1000	1.695	2.019	2.654	1.709	2.036	2.676	1.736	2.068	2.718
∞	1.645	1.960	2.576	1.645	1.960	2.576	1.645	1.960	2.576

Tables of Constants for Control charts

Table 8A - Variable Data

ref : AIAG manual for SPC

X bar and R Charts					X bar and s charts			
Chart for Averages	Chart for Ranges (R)				Chart for Averages	Chart for Standard Deviation (s)		
Control Limits Factor	Divisors to Estimate σ_x	Factors for Control Limits			Control Limits Factor	Divisors to estimate σ_x	Factors for Control Limits	
Subgroup size (n)	A_2	d_2	D_3	D_4	A_3	c_4	B_3	B_4
2	1.880	1.128	-	3.267	2.659	0.7979	-	3.267
3	1.023	1.693	-	2.574	1.954	0.8862	-	2.568
4	0.729	2.059	-	2.282	1.628	0.9213	-	2.266
5	0.577	2.326	-	2.114	1.427	0.9400	-	2.089
6	0.483	2.534	-	2.004	1.287	0.9515	0.030	1.970
7	0.419	2.704	0.076	1.924	1.182	0.9594	0.118	1.882
8	0.373	2.847	0.136	1.864	1.099	0.9650	0.185	1.815
9	0.337	2.970	0.184	1.816	1.032	0.9693	0.239	1.761
10	0.308	3.078	0.223	1.777	0.975	0.9727	0.284	1.716
15	0.223	3.472	0.347	1.653	0.789	0.9823	0.428	1.572
25	0.153	3.931	0.459	1.541	0.606	0.9896	0.565	1.435

	Centerline	Control Limits		σ_x
X bar and R Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$	$\frac{\bar{R}}{d_2}$
	$CL_R = \bar{R}$	$UCL_R = D_4 \bar{R}$	$LCL_R = D_3 \bar{R}$	
X bar and s Charts	$CL_{\bar{X}} = \bar{\bar{X}}$	$UCL_{\bar{X}} = \bar{\bar{X}} + A_3 \bar{s}$	$LCL_{\bar{X}} = \bar{\bar{X}} - A_3 \bar{s}$	$\frac{\bar{s}}{c_4}$
	$CL_s = \bar{s}$	$UCL_s = B_4 \bar{s}$	$LCL_s = B_3 \bar{s}$	